

## IIT-JEE-Mathematics-Mains-2005

### Mains

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1. A person goes to office either by car, scooter, bus or train probability of which being  $1/7, 3/7, 2/7$  and  $1/7$  respectively. Probability that he reaches office late, if he takes car, scooter, bus or train is  $2/9, 1/9, 4/9$  and  $1/9$  respectively. Given that he reaches office in time, then what is the probability that he travelled by a car?

2. Find the range of values of  $t$  for which

$$2 \sin t = (1-2x+5x^2)/(3x^2-2x-1), t \in [-\pi/2, \pi/2].$$

3. Circles with radii 3, 4 and 5 touch each other externally if P is the point of intersection of tangents to these circles at their points of contact. Find the distance of P from the point of contact.

4. Find the equation of the plane containing the line  $2x - y + z - 3 = 0$ ,  $3x + y + z = 5$  and at a distance of  $1/\sqrt{6}$  from the point  $(2, 1 - 1)$ .

5. If  $|f(x_1) - f(x_2)| < (x_1 - x_2)^2$ , for all  $x_1, x_2 \in \mathbb{R}$ . Find the equation of tangent to the curve  $y = f(x)$  at the point  $(1, 2)$ .

6. If total number of runs scored in  $n$  matches is  $((n+1)/4) (2^{n+1} - n - 2)$  where  $n > 1$ , and the runs scored in the  $k$ th match are given by  $k \cdot 2^{n+1-k}$ , where  $1 < k < n$ . Find  $n$ .

7. The area of the triangle formed by the intersection of a line parallel to  $x$ -axis and passing through  $(h, k)$  with the lines  $y = x$  and  $x + y = 2$  is  $4h^2$ . Find the locus of point P.

8. Evaluate

$$\int_0^{\pi} e^{|\cos x|} (2 \sin (1/2 \cos x) + 3 \cos (1/2 \cos x)) \sin x \, dx.$$

**9.** Incident ray is along the unit vector  $v$  and the reflected ray is along the unit vector  $w$ . The normal is along unit vector  $a$  outwards. Express vector  $w$  in terms of vector  $a$  and  $v$ .

**10.** Tangents are drawn from any point on the hyperbola  $x^2/9 - y^2/4 = 1$  to the circle  $x^2 + y^2 = 9$ . Find the locus of mid-point of the chord of contact.

**11.** Find the equation of the common tangent in 1st quadrant to the circle  $x^2 + y^2 = 16$  and the ellipse  $x^2/25 + y^2/4 = 1$ . Also find the length of the intercept of the tangent between the coordinate axes.

**12.** If length of tangent at any point on the curve  $y = f(x)$  intercepted between the point and the x-axis is of length 1. Find the equation of the curve.

**13.** Find the area bounded by the curves  $x^2 = y$ ,  $x^2 = -y$  and  $y^2 = 4x - 3$ .

**14.** If one of the vertices of the square circumscribing the circle  $|z - 1| = \sqrt{2}$  is  $2 + \sqrt{3}i$ . Find the other vertices of square.

**15.** If  $f(x - y) = f(x) \cdot g(y) - f(y) \cdot g(x)$  and  $g(x - y) = g(x) \cdot g(y) + f(x) \cdot f(y)$  for all  $x, y \in \mathbb{R}$ . If right hand derivative at  $x = 0$  exists for  $f(x)$ . Find derivative of  $g(x)$  at  $x = 0$ .

**16.** If  $p(x)$  be a polynomial of degree 3 satisfying  $p(-1) = 10$ ,  $p(1) = -6$  and  $p(x)$  has maximum at  $x = -1$  and  $p'(x)$  has minima at  $x = 1$ . Find the distance between the local maximum and local minimum of the curve.

**17.**  $f(x)$  is a differentiable function and  $g(x)$  is a double differentiable function such that  $|f(x)| < 1$  and  $f'(x) = g(x)$ . If  $f^2(0) + g^2(0) = 0$ . Prove that there exists some  $c \in (-3, 3)$  such that  $g(c) \cdot g''(c) < 0$ .

**18.** If

$$\begin{bmatrix} 4a^2 & 4a & 1 \\ 4b^2 & 4b & 1 \\ 4c^2 & 4c & 1 \end{bmatrix} \begin{bmatrix} f(-1) \\ f(1) \\ f(2) \end{bmatrix} = \begin{bmatrix} 3a^2 & + & 3a \\ 3b^2 & + & 3b \\ 3c^2 & + & 3c \end{bmatrix},$$

$f(x)$  is a quadratic function and its maximum value occurs at a point  $V$ .  $A$  is a point of intersection of  $y = f(x)$  with  $x$ -axis and point  $B$  is such that chord  $AB$  subtends a right angle at  $V$ . Find the area enclosed by  $f(x)$  and chord  $AB$ .